

Real-Time Welfare-Maximizing Regulation Allocation in Dynamic Aggregator-EVs System

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Abstract—The concept of vehicle-to-grid (V2G) has gained recent interest as more and more electric vehicles (EVs) are put to use. In this paper, we consider a dynamic aggregator-EVs system, where an aggregator centrally coordinates a large number of dynamic EVs to perform regulation service. We propose a Welfare-Maximizing Regulation Allocation (WMRA) algorithm for the aggregator to fairly allocate the regulation amount among its EVs. Compared to previous works, WMRA accommodates a wide spectrum of vital system characteristics, including dynamics of EV, limited EV battery size, EV battery degradation cost, and the cost of using external energy sources for the aggregator. The algorithm operates in real time and does not require any prior knowledge of the statistical information of the system. Theoretically, we demonstrate that WMRA is away from the optimum by $O(1/V)$, where V is a controlling parameter depending on EV's battery size. In addition, our simulation results indicate that WMRA can substantially outperform a suboptimal greedy algorithm.

Index Terms—Aggregator-EVs system; electric vehicles; real-time algorithm; V2G; welfare-maximizing regulation allocation.

I. INTRODUCTION

Electrification of personal transportation is expected to become prevalent in the near future. For example, millions of electric vehicles (EVs) will be operated in the United States by 2015 [1]. Besides serving the purpose of transportation, EVs can also be used as distributed electricity generation/storage devices when plugged-in [2]. Hence, the concept of vehicle-to-grid (V2G), referring to the integration of EVs with the power grid, has received increasing attention [2], [3].

Frequency regulation is a service to maintain the balance between power generation and load demand, which is vital to maintain the frequency of power grid at its nominal value. Traditionally, regulation service is achieved by turning on or off fast responsive generators and is the most expensive ancillary service [4]. Experiments show that EV's power electronics and battery can well respond to the frequent regulation signal. Thus it is possible to exploit plugged-in EV as a promising alternative to provide regulation service through charging/discharging, which potentially could reduce the cost of regulation service significantly [5]. However, since the regulation service is generally requested on the order of megawatts (MWs) while the power capacity of an EV is typically 5-20kW, it is often necessary for an aggregator

to coordinate a large number of EVs to provide regulation service [6]. In addition, frequent charging/discharging has direct effect on EV's battery life. Thus, it is important to design proper algorithm for regulation allocation in the aggregator-EVs system, especially in a real-time fashion.

There is a growing body of recent works on V2G regulation service. Specific to the aggregator-EVs system, which focuses on the interaction between the aggregator and EVs, centralized regulation allocation is studied in [7]–[11], where the objective is to maximize the profit of the aggregator or the EVs. In [7], a set of schemes based on different criteria of fairness among EVs are provided. In [8], the regulation allocation problem is formulated as quadratic programming. In [9], considering both regulation service and spinning reserves, the underlying problem is formulated as linear programming. In [10], the charging behaviour of EVs is also considered, so that the problem is then reduced to the control of the charging sequence and the charging rate of each EV, which is solved by dynamic programming. In [11], a real-time regulation control algorithm is proposed by formulating the problem as a Markov decision process, with the action space consisting of charging, discharging, and regulation. Finally, a distributed regulation allocation system is proposed in [12] using game theory, and a smart pricing policy is developed to incentivize EVs.

In addressing the regulation allocation problem, however, these earlier works have omitted to consider some essential characteristics of the aggregator-EVs system. For example, deterministic model is used in [7] and [10], which ignore the uncertainty of the system, e.g., the uncertainty of the electricity prices. The dynamics of the regulation signals is not incorporated in [12], nor the energy restriction of EV battery is considered. The self-charging/discharging activities in support of EV's own need are omitted in [7] and [12]. The potential cost of using external energy sources for the aggregator to accomplish regulation service is ignored in [7]–[11], and the cost of EV battery degradation due to frequent charging/discharging in performing regulation service is not considered in [8], [10]–[12].

In this work, we consider all of the above factors in a more complete aggregator-EVs system model, and develop a real-time algorithm for the aggregator to fairly allocate the regulation amount among the EVs. Specifically, considering an aggregator-EVs system providing long-term regulation service to a power grid, we aim to maximize the long-term social welfare of the aggregator-EVs system, under the long-term constraint on the battery degradation cost of each EV. To solve such a stochastic optimization problem, we adopt Lyapunov optimization technique, which is also used in [13]–[15] for

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demand side management in smart grid. We demonstrate how a solution to this maximization can be formulated under a general Lyapunov optimization framework [16], and propose a real-time allocation strategy specific to the aggregator-EVs system. The resultant Welfare-Maximizing Regulation Allocation (WMRA) algorithm does not require any statistical information of the system, and is shown to be asymptotically close to the optimum as EV's battery capacity increases. Finally, WMRA is compared to a greedy algorithm through simulation and is shown to offer substantial performance gains.

In our preliminary version of this work [17], the EVs are ideally assumed to be static, *i.e.*, they are in the aggregator-EVs system throughout the operational time. In this paper, to more realistically capture the dynamics of the aggregator-EVs system, we generalize the system model in [17] to accommodate dynamic EVs, which is considered in none of the previous works [7]–[12]. This generalization is challenging for the centralized control of regulation allocation, since the returning EV may have a different energy state compared to that when it leaves the system, and this energy difference will impose much more difficulties on the aggregator to handle EV's battery energy restriction under regulation service.

The rest of this paper is organized as follows. We describe the system model and formulate the regulation allocation problem in Section II. In Section III, we propose WMRA, and in Section IV we analyze its performance. Simulation results are exhibited in Section V, and we conclude in Section VI.

Notation: Denote $[a]^+$ as $\max[a, 0]$, $[a, b]^+$ as $\max[a, b]$, and $[a, b]^-$ as $\min[a, b]$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we propose a centralized dynamic aggregator-EVs system and formulate the regulation allocation problem mathematically.

A. Aggregator-EVs System and Regulation Service

Consider a time-slotted system with the time set $\mathcal{T} = \{0, 1, \dots\}$, where the regulation service is performed over equal time intervals of length Δt . At the beginning of each time slot t , the aggregator receives a random regulation signal G_t from the power grid. If $G_t > 0$ then the aggregator needs to perform *regulation down* by absorbing G_t units of energy from the power grid during time slot t , and if $G_t < 0$ then the aggregator needs to perform *regulation up* by contributing $|G_t|$ units of energy to the power grid during time slot t .

The aggregator coordinates N registered EVs to perform regulation service and can communicate with each EV bi-directionally when the EV is plugged-in. Assume that each EV can leave and re-join the system infinite times. For the i -th EV, denote $t_{ir,k} \in \mathcal{T}$ as its k -th returning time slot and $t_{il,k} \in \mathcal{T}$ as its k -th leaving time slot with $t_{ir,k} < t_{il,k}$, $\forall k \in \{1, 2, \dots\}$. Particularly, if the i -th EV is in the system at $t = 0$, let $t_{ir,1} = 0$. Define the set of the returning time slots for the i -th EV as $\mathcal{T}_{i,r} \triangleq \{t_{ir,1}, t_{ir,2}, \dots\}$, and the set of the leaving time slots as $\mathcal{T}_{i,l} \triangleq \{t_{il,1}, t_{il,2}, \dots\}$. Define

$$n_i(t) \triangleq \max \{k : t_{il,k} \leq t, k \in \{1, 2, \dots\}\} \quad (1)$$

as the total number of the times that the i -th EV leaves the system until time slot t . If such k does not exist, let $n_i(t) = 0$. Hence, from (1), we have $0 \leq n_i(t) < t$. From the assumption that the EV can leave and re-join the system infinite times, we have $\lim_{t \rightarrow \infty} n_i(t) = \infty$. Define

$$\mathcal{T}_{i,p} \triangleq \bigcup_{k=1}^{\infty} \{t_{ir,k}, t_{ir,k} + 1, \dots, t_{il,k+1} - 1\}$$

as the set containing all participating time slots of the i -th EV. In other words, the i -th EV is in the system for any $t \in \mathcal{T}_{i,p}$.

Define $\mathbf{1}_{i,t} \triangleq \begin{cases} 1, & \text{if } t \in \mathcal{T}_{i,p} \\ 0, & \text{otherwise} \end{cases}$, and $\mathbf{1}_t \triangleq [\mathbf{1}_{1,t}, \dots, \mathbf{1}_{N,t}]$.

At the beginning of each time slot, the aggregator allocates the required regulation energy amount $|G_t|$ among all *present* EVs. Denote $x_{id,t} \geq 0$ as the amount of regulation down energy allocated to the i -th EV, and $x_{iu,t} \geq 0$ as the amount of regulation up energy contributed by the i -th EV. Due to charging/discharging circuit limitation, assume that $x_{id,t}$ and $x_{iu,t}$ are upper bounded by $x_{i,\max} > 0$. If the i -th EV is not in the system at time slot t , then we have $x_{id,t} = x_{iu,t} = 0$. Define $\mathbf{x}_{d,t} \triangleq [x_{1d,t}, \dots, x_{Nd,t}]$ and $\mathbf{x}_{u,t} \triangleq [x_{1u,t}, \dots, x_{Nu,t}]$.

Assume that the i -th EV is in the system at some $t \in \mathcal{T}_{i,p}$. Denote $s_{i,t}$ as its energy state at the beginning of time slot t , restricted by $0 \leq s_{i,t} \leq s_{i,\text{cap}}$, where $s_{i,\text{cap}}$ is the battery capacity of the EV. After regulation service at time slot t , the energy state of the i -th EV at time slot $t + 1$ is given by

$$s_{i,t+1} = s_{i,t} + \mathbf{1}_{d,t}x_{id,t} - \mathbf{1}_{u,t}x_{iu,t} = s_{i,t} + b_{i,t}, \quad (2)$$

where $\mathbf{1}_{d,t} \triangleq \begin{cases} 1, & \text{if } G_t > 0 \\ 0, & \text{otherwise} \end{cases}$, $\mathbf{1}_{u,t} \triangleq \begin{cases} 1, & \text{if } G_t < 0 \\ 0, & \text{otherwise} \end{cases}$, and $b_{i,t} \triangleq \mathbf{1}_{d,t}x_{id,t} - \mathbf{1}_{u,t}x_{iu,t}$. Note that $\mathbf{1}_{d,t}\mathbf{1}_{u,t} = 0, \forall t$, since regulation down and up services cannot happen at the same time. Charging a battery to near its capacity or discharging it to close to the zero energy state can significantly reduce battery's lifetime [18]. Therefore, lower and upper bounds on the battery energy state are usually imposed by its manufacturer or user. Denote $[s_{i,\min}, s_{i,\max}]$ as the preferred energy range of the i -th EV with $0 \leq s_{i,\min} < s_{i,\max} \leq s_{i,\text{cap}}$. By such constraint, the resultant energy state at time slot $t + 1$ should satisfy which means that the allocated regulation amounts $x_{id,t}$ and $x_{iu,t}$ for the i -th EV should satisfy $0 \leq x_{id,t} \leq h_{id,t}$, and $0 \leq x_{iu,t} \leq h_{iu,t}$, respectively, where

$$h_{id,t} \triangleq \begin{cases} [x_{i,\max}, s_{i,\max} - s_{i,t}]^-, & \text{if } \mathbf{1}_{i,t} = 1 \\ 0 & \text{otherwise} \end{cases},$$

and

$$h_{iu,t} \triangleq \begin{cases} [x_{i,\max}, s_{i,t} - s_{i,\min}]^-, & \text{if } \mathbf{1}_{i,t} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

From time to time, the i -th EV may need to stop its regulation service and leave the system for personal reason or self-charging/discharging purposes. When the EV is out of the system, it cannot perform regulation service and the aggregator has no information of its energy state. When returning, the EV may have a different energy state compared to its last leaving energy state. Assume that all returning energy states of the i -th EV lie in the preferred energy range,

i.e., $s_{i,\min} \leq s_{i,t} \leq s_{i,\max}, \forall t \in \mathcal{T}_{i,r}$, by the EV's self-control. Further, define

$$\Delta_{i,k} \triangleq s_{i,t_{il,k}} - s_{i,t_{ir,k+1}}, \forall k \in \{1, 2, \dots\}$$

as the difference between the EV's k -th leaving energy state and $(k+1)$ -th returning energy state, and assume that

$$\lim_{K \rightarrow \infty} \mathbb{E} \left[\left| \frac{1}{K} \sum_{k=1}^K \Delta_{i,k} \right| \right] = 0.$$

It can be shown that such condition holds if $\Delta_{i,k}$ is i.i.d. with mean zero and finite variance, which is a rather mild assumption considering the random behaviour of each EV.

For each EV, the regulation service gain comes at the cost of battery degradation due to frequent charging/discharging activities. Denote $C_i(x)$ as the degradation cost function of the regulation amount x for the i -th EV, with $0 \leq C_i(x) \leq c_{i,\max}$ and $C_i(0) = 0$. Since faster charging or discharging, *i.e.*, larger value of $x_{id,t}$ or $x_{iu,t}$, has a more detrimental effect on the battery's lifetime, we assume $C_i(x)$ to be convex, continuous, and non-decreasing. We further assume that each EV imposes an upper bound $c_{i,\text{up}}$, $0 \leq c_{i,\text{up}} \leq c_{i,\max}$, on the time-averaged battery degradation, expressed by $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} [\mathbf{1}_{d,t} C_i(x_{id,t}) + \mathbf{1}_{u,t} C_i(x_{iu,t})] \leq c_{i,\text{up}}$.

Finally, the total regulation amount provided by the EVs may not be sufficient to serve the requested regulation amount. For brevity, define

$$x_{i,t} \triangleq \mathbf{1}_{d,t} x_{id,t} + \mathbf{1}_{u,t} x_{iu,t} \geq 0$$

as the regulation amount allocated to the i -th EV at time slot t . Then, the insufficiency of regulation amount means that $\sum_{i=1}^N x_{i,t} < |G_t|$ for regulation down or up. This could be due to, for example, a lack of participating EVs, or high cost of battery degradation. The gap between $\sum_{i=1}^N x_{i,t}$ and $|G_t|$ represents an energy surplus in the case of regulation down, or an energy deficit in the case of regulation up. Such surplus or deficit must be cleared, or the regulation service fails. Therefore, from time to time, the aggregator may need to exploit more expensive external energy sources, such as from the traditional regulation market. Denote the unit costs of clearing energy surplus and energy deficit at time slot t as $e_{s,t}$ and $e_{d,t}$, respectively, which are both random but restricted in $[e_{\min}, e_{\max}]$. Then, the cost for the aggregator at time slot t is

$$e_t \triangleq \mathbf{1}_{d,t} e_{s,t} \left(G_t - \sum_{i=1}^N x_{id,t} \right) + \mathbf{1}_{u,t} e_{d,t} \left(|G_t| - \sum_{i=1}^N x_{iu,t} \right).$$

B. Fair Regulation Allocation through Welfare Maximization

The objective of the aggregator is to maximize the long-term social welfare of the aggregator-EVs system, *i.e.*, to fairly allocate the regulation amount among the EVs, while respecting EVs' battery degradation constraints and reducing the need to utilize the expensive external energy sources. To this end, we formulate the regulation allocation problem as the

following stochastic optimization problem:

P1:

$$\max_{\mathbf{x}_{d,t}, \mathbf{x}_{u,t}} \sum_{i=1}^N \omega_i U \left(\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[x_{i,t}] \right) - \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[e_t]$$

$$\text{s.t. } 0 \leq x_{id,t} \leq h_{id,t}, \forall i, \quad (3)$$

$$0 \leq x_{iu,t} \leq h_{iu,t}, \forall i, \quad (4)$$

$$\sum_{i=1}^N x_{id,t} \leq \mathbf{1}_{d,t} G_t, \quad (5)$$

$$\sum_{i=1}^N x_{iu,t} \leq \mathbf{1}_{u,t} |G_t|, \quad (6)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} [\mathbf{1}_{d,t} C_i(x_{id,t}) + \mathbf{1}_{u,t} C_i(x_{iu,t})] \leq c_{i,\text{up}}, \forall i, \quad (7)$$

where $\omega_i > 0$ is the normalized weight associated with the i -th EV, and $U(\cdot)$ is a utility function assumed to be concave, continuous, and non-decreasing, with a domain bounded within $[0, x_{i,\max}]$, $\forall i$. Furthermore, to facilitate later analysis, we make a mild assumption that the utility function $U(\cdot)$ satisfies

$$U(x) \leq U(0) + \mu x, \forall x \in \left[0, \max_{1 \leq i \leq N} \{x_{i,\max}\} \right], \quad (8)$$

where $\mu > 0$. One sufficient condition for (8) to hold is that $U(\cdot)$ has finite positive derivate at zero, such as $U(x) = \log(1+x)$. The expectations in the above optimization problem are taken over the randomness of the system inputs.

Remarks: In the objective function of **P1**, the first term considers each EV's welfare under the utility function $U(\cdot)$ and the weight ω_i , and the second term reflects the aggregator's cost, which is affected by the regulation amounts of all EVs. In (3) and (4), for each EV, hard constraint is set for the regulation amount at each time slot, while in (7), long-term average constraint on the battery degradation cost due to regulation allocation is set. Note that constraints (5) and (6) ensure that $x_{id,t} = 0$ for regulation up service ($G_t < 0$), and $x_{iu,t} = 0$ for regulation down service ($G_t > 0$). These two constraints couple the regulation amounts of all EVs.

III. WELFARE-MAXIMIZING REGULATION ALLOCATION

In this section, we first apply a sequence of two reformulations to **P1**, then propose a real-time welfare-maximizing regulation allocation (WMRA) algorithm to solve the resultant optimization problem. The performance analysis of the proposed WMRA will be shown in Section IV.

A. Problem Transformation

The objective of **P1** contains a function of long-term average, which complicates the problem. However, in general, such a problem can be converted to a problem of maximizing a long-term average of the function [16]. Specifically, we transform **P1** as follows.

We first introduce an auxiliary N -dimensional vector $\mathbf{z}_t \triangleq [z_{1,t}, \dots, z_{N,t}]$ with the constraints

$$0 \leq z_{i,t} \leq x_{i,\max}, \forall i, \text{ and} \quad (9)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[z_{i,t}] = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[x_{i,t}], \forall i. \quad (10)$$

From the above constraints, the auxiliary variable $z_{i,t}$ and the regulation allocation amount $x_{i,t}$ are within the same range and have the same long-term average behaviour. We now consider the following problem.

P2:

$$\begin{aligned} \max_{\mathbf{x}_{d,t}, \mathbf{x}_{u,t}, \mathbf{z}_t} \quad & \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \left[\left(\sum_{i=1}^N \omega_i U(z_{i,t}) \right) - e_t \right] \\ \text{s.t.} \quad & (3), (4), (5), (6), (7), (9), \text{ and } (10). \end{aligned}$$

Compared to **P1**, the optimization in **P2** is over $\mathbf{x}_{d,t}$, $\mathbf{x}_{u,t}$ and \mathbf{z}_t with two more constraints (9) and (10). Note that **P2** contains no function of time average; instead, it maximizes a long-term time average of the expected social welfare.

Denote $(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}, \mathbf{z}_t^{\text{opt}})$ as an optimal solution to **P1**, and $(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*, \mathbf{z}_t^*)$ as an optimal solution to **P2**. Define $\bar{\mathbf{z}}_t^{\text{opt}} \triangleq [\bar{z}_{1,t}^{\text{opt}}, \dots, \bar{z}_{N,t}^{\text{opt}}]$ with

$$\bar{z}_{i,t}^{\text{opt}} \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[x_{i,t}^{\text{opt}}], \quad \forall i.$$

Denote the objective functions of **P1** and **P2** as $f_1(\cdot)$ and $f_2(\cdot)$, respectively. The equivalence of **P1** and **P2** is stated below.

Lemma 1: **P1** and **P2** have the same optimal objective, i.e., $f_1(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}) = f_2(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*, \mathbf{z}_t^*)$. Furthermore, $(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}, \bar{\mathbf{z}}_t^{\text{opt}})$ is an optimal solution to **P2**, and $(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*)$ is an optimal solution to **P1**.

Proof: The proof follows the general framework given in [16]. Details specific to our system are given in Appendix A. ■

Lemma 1 indicates that the transformation from **P1** to **P2** results in no loss of optimality. Thus, in the following, we will focus on solving **P2** instead.

B. Problem Relaxation

P2 is still a challenging problem to solve since in constraints (3) and (4), the regulation allocation amount of each EV may depend on its current energy state $s_{i,t}$, which couples with all previous regulation allocation amounts. To avoid such coupling, we relax the constraints of $x_{id,t}$ and $x_{iu,t}$ and introduce **P3** below.

P3:

$$\begin{aligned} \max_{\mathbf{x}_{d,t}, \mathbf{x}_{u,t}, \mathbf{z}_t} \quad & \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \left[\left(\sum_{i=1}^N \omega_i U(z_{i,t}) \right) - e_t \right] \\ \text{s.t.} \quad & 0 \leq x_{id,t} \leq \mathbf{1}_{i,t} x_{i,\max}, \forall i, \quad (11) \\ & 0 \leq x_{iu,t} \leq \mathbf{1}_{i,t} x_{i,\max}, \forall i, \quad (12) \end{aligned}$$

$$\begin{aligned} & \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[b_{i,t}] = 0, \forall i, \quad (13) \\ & (5), (6), (7), (9), \text{ and } (10), \end{aligned}$$

where in (13) $b_{i,t}$ is defined below (2). In **P3**, we have replaced the constraints (3) and (4) in **P2** with (11)–(13), thus have removed the dependency on the current energy state $s_{i,t}$. We next demonstrate that, any $(\mathbf{x}_{d,t}, \mathbf{x}_{u,t})$ that meets (3) and (4) also satisfies (11)–(13), i.e., any feasible solution of **P2** is also feasible for **P3**.

Considering the i -th EV, the constraints (3) and (4) in **P2** are equivalent to the following two sub-constraints:

if $\mathbf{1}_{i,t} = 1$, then

$$0 \leq x_{id,t} \leq x_{i,\max} \quad (14)$$

$$0 \leq x_{iu,t} \leq x_{i,\max} \quad (15)$$

$$s_{i,\min} \leq s_{i,t+1} \leq s_{i,\max}; \quad (16)$$

if $\mathbf{1}_{i,t} = 0$, then

$$x_{id,t} = x_{iu,t} = 0. \quad (17)$$

Since $s_{i,t}$ is also bounded for any returning time slot $t \in \mathcal{T}_{i,r}$, together with (16), we have $s_{i,\min} \leq s_{i,t} \leq s_{i,\max}, \forall t \in \mathcal{T}_{i,p} \cup \mathcal{T}_{i,l}$. Note that (14), (15), and (17) imply (11) and (12), so we are left to justify that the boundedness of $s_{i,t}$ implies (13).

Lemma 2: For the i -th EV, assume that $\lim_{K \rightarrow \infty} \mathbb{E} \left[\left| \frac{1}{K} \sum_{k=1}^K \Delta_{i,k} \right| \right] = 0$ and $\lim_{t \rightarrow \infty} n_i(t) = \infty$. If $s_{i,\min} \leq s_{i,t} \leq s_{i,\max}, \forall t \in \mathcal{T}_{i,p} \cup \mathcal{T}_{i,l}$, then the constraint (13) holds, i.e., $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[b_{i,t}] = 0$.

Proof: See Appendix B. ■

From Lemma 2, we know that, the boundedness of $s_{i,t}$ indeed implies (13), which completes our demonstration that **P3** is a relaxed version of **P2** with larger feasible solution set. We will later show that our proposed real-time algorithm for **P3** ensures (3) and (4) are always satisfied, and thus provides a feasible solution to **P2** and to the original problem **P1**.

The relaxed problem **P3** allows us to apply Lyapunov optimization to design a real-time algorithm for solving welfare maximization. This relaxation technique to accommodate the type of time-coupled action constraints such as (3) and (4) is first introduced in [19] for a power-cost minimization problem in data centers equipped with stored energy. Unlike in [19], the structure of our problem is more complicated, where the dynamics of the distributed storages (EVs) are considered as well as a nonlinear objective which allows both positive and negative values for the energy requirement G_t . Thus, the algorithm design is more involved to ensure that the original constraints in **P2** are satisfied.

C. WMRA Algorithm

In this subsection, We propose a WMRA algorithm to solve **P3** by employing Lyapunov optimization technique.

We first define three virtual queues for each EV with the associated queue backlogs $J_{i,t}$, $H_{i,t}$, and $K_{i,t}$. The evolutionary behaviours of $J_{i,t}$, $H_{i,t}$, and $K_{i,t}$, $\forall i$, are as follows.

$$J_{i,t+1} = [J_{i,t} + \mathbf{1}_{d,t} C_i(x_{id,t}) + \mathbf{1}_{u,t} C_i(x_{iu,t}) - c_{i,\text{up}}]^+. \quad (18)$$

$$H_{i,t+1} = H_{i,t} + z_{i,t} - x_{i,t}. \quad (19)$$

$$K_{i,t} = \begin{cases} s_{i,t} - c_i, & \text{if } t \in \mathcal{T}_{i,r} \\ K_{i,t-1} + b_{i,t-1}, & \text{otherwise,} \end{cases} \quad (20)$$

where in (20) we design the constant $c_i \triangleq s_{i,\min} + 2x_{i,\max} + V(\omega_i\mu + e_{\max})$ with $V \in [0, V_{\max}]$ and

$$V_{\max} \triangleq \min_{1 \leq i \leq N} \left\{ \frac{s_{i,\max} - s_{i,\min} - 4x_{i,\max}}{2(\omega_i\mu + e_{\max})} \right\}. \quad (21)$$

The role of V will be explained later. It will also be clear that the specific expressions of c_i and V_{\max} are in fact to ensure the boundedness of the energy state $s_{i,t}$. Note that $x_{i,\max}$ is generally much smaller than the energy capacity. For example, for Tesla Model S [20], the energy capacity is 40kWh, and $x_{i,\max} = 0.83\text{kWh}$ if the maximum charging rate is applied and the regulation duration is 5 minutes. Therefore, generally we always have $V_{\max} > 0$.

From (20), $K_{i,t}$ is re-initialized as a shifted version of $s_{i,t}$ every time the i -th EV returning to the aggregator-EVs system; also, $K_{i,t}$ has the same evolutionary behaviour as $s_{i,t}$ for $t \in \mathcal{T}_{i,p} \cup \mathcal{T}_{i,l}$. Therefore, (20) implies that $K_{i,t} = s_{i,t} - c_i$, $\forall t \in \mathcal{T}_{i,p} \cup \mathcal{T}_{i,l}$. In addition, since $b_{i,t} = 0$ when $\mathbf{1}_{i,t} = 0$, we have

$$K_{i,t} = K_{i,t_{il,k}}, \quad \forall t \in \{t_{il,k}, \dots, t_{ir,k+1} - 1\}, \\ \text{and } \forall k \in \{1, 2, \dots\}.$$

By introducing the virtual queues, the constraints (7) and (10) hold if the queues $J_{i,t}$ and $H_{i,t}$ are mean rate stable, respectively [16]. Below we give the definition of mean rate stability of a queue.

Definition: A discrete time process $Q(t)$ is mean rate stable if $\lim_{t \rightarrow \infty} \frac{\mathbb{E}[|Q(t)|]}{t} = 0$.

Unlike $J_{i,t}$ and $H_{i,t}$, since $K_{i,t}$ is re-initialized when $t \in \mathcal{T}_{i,r}$, a new virtual queue is essentially created every time the i -th EV re-joining the system. Therefore, the mean rate stability of $K_{i,t}$ is not sufficient for the constraint (13) to hold, and stronger condition is required. For now, since $K_{i,t}$ is just a shifted version of $s_{i,t}$ for $t \in \mathcal{T}_{i,p} \cup \mathcal{T}_{i,l}$, from Lemma 2, the following lemma is straightforward.

Lemma 3: For the i -th EV, assume that $\lim_{K \rightarrow \infty} \mathbb{E} \left[\left| \frac{1}{K} \sum_{k=1}^K \Delta_{i,k} \right| \right] = 0$ and $\lim_{t \rightarrow \infty} n_i(t) = \infty$. If $s_{i,\min} - c_i \leq K_{i,t} \leq s_{i,\max} - c_i$, $\forall t \in \mathcal{T}_{i,p} \cup \mathcal{T}_{i,l}$, then the constraint (13) holds, i.e., $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[b_{i,t}] = 0$.

Later we will show that by our proposed algorithm, the boundedness condition of $K_{i,t}$ in Lemma 3 can be guaranteed.

Define $\mathbf{J}_t \triangleq [J_{1,t}, \dots, J_{N,t}]$, $\mathbf{H}_t \triangleq [H_{1,t}, \dots, H_{N,t}]$, $\mathbf{K}_t \triangleq [K_{1,t}, \dots, K_{N,t}]$, and $\mathbf{\Theta}_t \triangleq [\mathbf{H}_t, \mathbf{J}_t, \mathbf{K}_t]$ with the initial value $\mathbf{\Theta}_0 = \mathbf{0}$. Define the Lyapunov function $L(\mathbf{\Theta}_t) \triangleq \frac{1}{2} \sum_{i=1}^N (H_{i,t}^2 + J_{i,t}^2 + K_{i,t}^2)$, and the associated one-slot Lyapunov drift as $\Delta(\mathbf{\Theta}_t) \triangleq \mathbb{E}[L(\mathbf{\Theta}_{t+1}) - L(\mathbf{\Theta}_t) | \mathbf{\Theta}_t]$. Let the drift-minus-welfare function be $\Delta(\mathbf{\Theta}_t) - V \mathbb{E} \left[\sum_{i=1}^N \omega_i U(z_{i,t}) - e_t | \mathbf{\Theta}_t \right]$, where $V \in [0, V_{\max}]$ is the weight associated with the welfare objective. Therefore, the larger V , the more weight is put on the welfare objective in the drift-minus-welfare function. We give an upper bound on the drift-minus-welfare function in the following proposition.

Proposition 1: The drift-minus-welfare function is upper-

bounded as

$$\begin{aligned} \Delta(\mathbf{\Theta}_t) - V \mathbb{E} \left[\sum_{i=1}^N \omega_i U(z_{i,t}) - e_t | \mathbf{\Theta}_t \right] \\ \leq B + \sum_{i=1}^N K_{i,t} \mathbb{E}[b_{i,t} | \mathbf{\Theta}_t] + \sum_{i=1}^N H_{i,t} \mathbb{E}[z_{i,t} - x_{i,t} | \mathbf{\Theta}_t] \\ + \sum_{i=1}^N J_{i,t} \mathbb{E}[\mathbf{1}_{d,t} C_i(x_{id,t}) + \mathbf{1}_{u,t} C_i(x_{iu,t}) - c_{i,up} | \mathbf{\Theta}_t] \\ - V \mathbb{E} \left[\sum_{i=1}^N \omega_i U(z_{i,t}) - e_t | \mathbf{\Theta}_t \right], \end{aligned} \quad (22)$$

where $B \triangleq \frac{1}{2} \sum_{i=1}^N [x_{i,\max}^2 + [x_{i,\max}^2, c_i^2]^+ + [c_{i,up}^2, (c_{i,\max} - c_{i,up})^2]^+]$ and $V \in [0, V_{\max}]$.

Proof: See Appendix C. ■

We now propose the WMRA algorithm that minimizes the upper bound on the drift-minus-welfare function in (22) at each time slot. This is equivalent to solving the following decoupled sub-problems with respect to \mathbf{z}_t , $\mathbf{x}_{d,t}$, and $\mathbf{x}_{u,t}$, separately. Denote the auxiliary vector and the allocated regulation down and up energy amount vectors produced by WMRA as $\tilde{\mathbf{z}}_t \triangleq [\tilde{z}_{1,t}, \dots, \tilde{z}_{N,t}]$, $\tilde{\mathbf{x}}_{d,t} \triangleq [\tilde{x}_{1d,t}, \dots, \tilde{x}_{Nd,t}]$, and $\tilde{\mathbf{x}}_{u,t} \triangleq [\tilde{x}_{1u,t}, \dots, \tilde{x}_{Nu,t}]$, respectively. Specifically, we obtain $\tilde{z}_{i,t}, \forall i$, by solving (a):

$$(a): \min_{z_{i,t}} H_{i,t} z_{i,t} - \omega_i V U(z_{i,t}) \quad \text{s.t. } 0 \leq z_{i,t} \leq x_{i,\max}.$$

For $G_t > 0$, we obtain $\tilde{\mathbf{x}}_{d,t}$ by solving (b1):

$$\begin{aligned} (b1): \min_{\mathbf{x}_{d,t}} V e_{s,t}(G_t - \sum_{i=1}^N x_{id,t}) - \sum_{i=1}^N H_{i,t} x_{id,t} \\ + \sum_{i=1}^N J_{i,t} C_i(x_{id,t}) + \sum_{i=1}^N K_{i,t} x_{id,t} \\ \text{s.t. } 0 \leq x_{id,t} \leq \mathbf{1}_{i,t} x_{i,\max}, \quad \sum_{i=1}^N x_{id,t} \leq G_t. \end{aligned}$$

For $G_t < 0$, we obtain $\tilde{\mathbf{x}}_{u,t}$ by solving (b2):

$$\begin{aligned} (b2): \min_{\mathbf{x}_{u,t}} V e_{d,t}(|G_t| - \sum_{i=1}^N x_{iu,t}) - \sum_{i=1}^N H_{i,t} x_{iu,t} \\ + \sum_{i=1}^N J_{i,t} C_i(x_{iu,t}) - \sum_{i=1}^N K_{i,t} x_{iu,t} \\ \text{s.t. } 0 \leq x_{iu,t} \leq \mathbf{1}_{i,t} x_{i,\max}, \quad \sum_{i=1}^N x_{iu,t} \leq |G_t|. \end{aligned}$$

Note that (a), (b1), and (b2) are all convex problems, so they can be efficiently solved using standard methods such as the interior point method and the Lagrange dual method [21]. We summarize WMRA in Algorithm 1. Note from Steps (2b) and (2c) that, the solutions of (a) and (b1) (or (b2)) affect each other over multiple time slots through the update of $H_{i,t}, \forall i$. To perform WMRA, no statistical information of the system is needed, which makes the algorithm easy to implement.

Algorithm 1 Welfare-Maximizing Regulation Allocation (WMRA) Algorithm.

- 1: The aggregator initializes the virtual queue vector $\Theta_0 = \mathbf{0}$, and re-initialize $K_{i,t} = s_{i,t} - c_i$ for $t \in \mathcal{T}_{i,r}, \forall i$.
 - 2: At the beginning of each time slot t , the aggregator performs the following steps sequentially.
 - (2a) Observe $G_t, e_{s,t}, e_{d,t}, \mathbf{1}_t$ (if $\mathbf{1}_t$ cannot be predicted), $\mathbf{J}_t, \mathbf{H}_t$, and \mathbf{K}_t .
 - (2b) Solve (a) and record an optimal solution $\tilde{\mathbf{z}}_t$. If $G_t > 0$, solve (b1) and record an optimal solution $\tilde{\mathbf{x}}_{d,t}$. If $G_t < 0$, solve (b2) and record an optimal solution $\tilde{\mathbf{x}}_{u,t}$. Allocate the regulation amounts based on $\tilde{\mathbf{x}}_{d,t}$ and $\tilde{\mathbf{x}}_{u,t}$. If $\sum_{i=1}^N \tilde{x}_{id,t} < G_t$ or $\sum_{i=1}^N \tilde{x}_{iu,t} < |G_t|$, then clear the imbalance using the external energy.
 - (2c) Update the virtual queues $J_{i,t}, H_{i,t}$, and $K_{i,t}, \forall i$, based on (18), (19), and (20), respectively.
-

IV. PERFORMANCE ANALYSIS

In this section, we characterize the performance of WMRA with respect to our original problem **P1**.

A. Properties of WMRA Algorithm

We now show that WMRA can ensure the boundedness of each EV's energy state. The following lemma characterizes sufficient conditions under which the solution of $\tilde{x}_{id,t}$ and $\tilde{x}_{iu,t}$ under WMRA is zero.

Lemma 4: Under the WMRA algorithm, for any $t \in \mathcal{T}_{i,p}$,

- 1) for $G_t > 0$, if $K_{i,t} > x_{i,\max} + V(\omega_i \mu + e_{\max})$, then $\tilde{x}_{id,t} = 0$, which means that $K_{i,t+1}$ cannot be increased at the next time slot; and
- 2) for $G_t < 0$, if $K_{i,t} < -x_{i,\max} - V(\omega_i \mu + e_{\max})$, then $\tilde{x}_{iu,t} = 0$, which means that $K_{i,t+1}$ cannot be decreased at the next time slot.

Proof: See Appendix D. ■

Since Lemma 4 on the other hand provides conditions under which queue backlog $K_{i,t}$ can no longer increase or decrease, using Lemma 4, we can prove the boundedness of $K_{i,t}$ below.

Lemma 5: Under the WMRA algorithm, queue backlog $K_{i,t}$ associated with the i -th EV is bounded by $s_{i,\min} - c_i \leq K_{i,t} \leq s_{i,\max} - c_i, \forall t \in \mathcal{T}_{i,p} \cup \mathcal{T}_{i,l}$.

Proof: Consider the set $\{t_{ir,k}, t_{ir,k}+1, \dots, t_{il,k}\}$ for any $k \in \{1, 2, \dots\}$. We show below that $K_{i,t}$ is bounded for any t in such set by induction.

First consider the upper bound. For the time slot $t_{ir,k}$, based on (20) and $s_{i,t_{ir,k}} \leq s_{i,\max}$, there is $K_{i,t_{ir,k}} \leq s_{i,\max} - c_i$. Assume that the upper bound holds for time slot t and consider the following two cases of $K_{i,t}$.

Case 1: $x_{i,\max} + V(\omega_i \mu + e_{\max}) < K_{i,t} \leq s_{i,\max} - c_i$ (We can check that $x_{i,\max} + V(\omega_i \mu + e_{\max}) < s_{i,\max} - c_i$ since $V \leq V_{\max}$). For $G_t > 0$, from Lemma 4 1), there is $\tilde{x}_{id,t} = 0$. Therefore, $K_{i,t+1} = K_{i,t} \leq s_{i,\max} - c_i$. For $G_t < 0$, we have $K_{i,t+1} = K_{i,t} - x_{iu,t} \leq K_{i,t} \leq s_{i,\max} - c_i$.

Case 2: $K_{i,t} \leq x_{i,\max} + V(\omega_i \mu + e_{\max})$. From (20), $K_{i,t+1} \leq 2x_{i,\max} + V(\omega_i \mu + e_{\max}) \leq s_{i,\max} - c_i$, where the last inequality holds since $V \leq V_{\max}$.

Now look at the lower bound. For the time slot $t_{ir,k}$, based on (20) and $s_{i,t_{ir,k}} \geq s_{i,\min}$, there is $K_{i,t_{ir,k}} \geq s_{i,\min} - c_i$. Assume that the lower bound holds for time slot t and consider the following two cases of $K_{i,t}$.

Case 1': $s_{i,\min} - c_i \leq K_{i,t} < -x_{i,\max} - V(\omega_i \mu + e_{\max})$ (We can check that $s_{i,\min} - c_i < -x_{i,\max} - V(\omega_i \mu + e_{\max})$ since $x_{i,\max} > 0$). For $G_t < 0$, from Lemma 4 2), there is $\tilde{x}_{iu,t} = 0$. Therefore, $K_{i,t+1} = K_{i,t} \geq s_{i,\min} - c_i$. For $G_t > 0$, we have $K_{i,t+1} = K_{i,t} + x_{id,t} \geq K_{i,t} \geq s_{i,\min} - c_i$.

Case 2': $K_{i,t} \geq -x_{i,\max} - V(\omega_i \mu + e_{\max})$. From (20), $K_{i,t+1} \geq -2x_{i,\max} - V(\omega_i \mu + e_{\max})$, which is exactly $s_{i,\min} - c_i$. ■

Remarks: To track the energy state $s_{i,t}$, in principle, the shift c_i can be any number. However, to ensure the boundedness of $K_{i,t}$, the form of c_i is uniquely determined from the proof of Case 2'. For the design of V_{\max} , to make the proof in Case 1 work, it is sufficient to let $V_{\max} = \min_{1 \leq i \leq N} \left\{ \frac{s_{i,\max} - s_{i,\min} - 3x_{i,\max} - \epsilon}{2(\omega_i \mu + e_{\max})} \right\}$ where $\epsilon > 0$ can be arbitrarily small. This ϵ is further determined as $x_{i,\max}$ based on the proof in Case 2.

Note that Lemma 5 is a sample path result. Therefore, it is true regardless of the statistics of the system. In addition, note that since the boundedness condition of $K_{i,t}$ in Lemma 3 is now satisfied, the conclusion there is true under WMRA. Recall that $K_{i,t} = s_{i,t} - c_i$ for any $t \in \mathcal{T}_{ip} \cup \mathcal{T}_{il}$. Using Lemma 5, the following lemma is straightforward.

Lemma 6: Under the WMRA algorithm, the energy state of the i -th EV is bounded by $s_{i,\min} \leq s_{i,t} \leq s_{i,\max}, \forall t \in \mathcal{T}_{i,p} \cup \mathcal{T}_{i,l}$.

Hence, from Lemma 6, the constraints (3) and (4) in **P2** are met under WMRA.

B. Optimality of WMRA Algorithm

In this subsection, we investigate the optimality of WMRA by considering EVs with both predictable and random dynamics, which are described below.

- 1) EVs with predictable dynamics: Predictable dynamics could happen when each EV joins and leaves the aggregator-EVs system regularly (e.g. from 9am to 12pm in the morning, then from 2pm to 6pm in the afternoon). Therefore, an EV's leaving and returning time slots can be predicted by the aggregator, which means that, the aggregator is aware of the realization of $\mathbf{1}_t, \forall t$, in advance, and it does not have to observe $\mathbf{1}_t$ every time slot. In this case, the system state at time slot t is defined as $A_t \triangleq (G_t, e_{s,t}, e_{d,t})$.
- 2) EVs with random dynamics: If the EVs do not participate in the aggregator-EVs system regularly, then the aggregator cannot predict their dynamics beforehand, and therefore, has to observe $\mathbf{1}_t$ every time slot. In this case, the system state at time slot t is defined as $A_t \triangleq (G_t, e_{s,t}, e_{d,t}, \mathbf{1}_t)$.

Note that the WMRA algorithm is the same under both of the above cases. The only difference between them is that, in the optimization problem **P3**, the expectations are taken over different randomness of the system state. The performance under WMRA as compared to the optimal solution of **P1**

is given in the following theorem, which applies to both predictable and random dynamics.

Theorem 1: Given the system state A_t is i.i.d. over time,

- 1) $(\tilde{\mathbf{x}}_{d,t}, \tilde{\mathbf{x}}_{u,t})$ is feasible for **P1**, i.e., it satisfies (3), (4), (5), (6), and (7).

- 2) $f_1(\tilde{\mathbf{x}}_{d,t}, \tilde{\mathbf{x}}_{u,t}) \geq f_1(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}) - \frac{B}{V}$,

where $B \triangleq \frac{1}{2} \sum_{i=1}^N \left[x_{i,\max}^2 + [x_{i,\max}^2, c_i^2]^+ + [c_{i,\text{up}}^2, (c_{i,\max} - c_{i,\text{up}})^2]^+ \right]$ and $V \in [0, V_{\max}]$.

Proof: See Appendix E. ■

Remarks: From Theorem 1, the welfare performance of WMRA is away from the optimum by $O(1/V)$. Hence, the larger V , the better the performance of WMRA. However, in practice, due to the boundedness condition of EV's battery capacity, V cannot be arbitrarily large and is upper bounded by V_{\max} , which is defined in (21). Note that V_{\max} increases with the smallest span of the EVs' preferred battery capacity ranges, i.e., $\min_{1 \leq i \leq N} \{s_{i,\max} - s_{i,\min}\}$. Therefore, roughly speaking, the performance gap between WMRA and the optimum decreases as the smallest battery capacity increases. Asymptotically, as the EVs' battery capacities go to infinity, WMRA would achieve exactly the optimum.

In the Theorem 1, the i.i.d. condition of A_t can be relaxed to Markovian over time, and a similar performance bound can be obtained.

Theorem 2: Given that the system state A_t evolves based on a finite state irreducible and aperiodic Markov chain,

- 1) $(\tilde{\mathbf{x}}_{d,t}, \tilde{\mathbf{x}}_{u,t})$ is feasible for **P1**, i.e., it satisfies (3), (4), (5), (6), and (7).

- 2) $f_1(\tilde{\mathbf{x}}_{d,t}, \tilde{\mathbf{x}}_{u,t}) \geq f_1(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}) - O(1/V)$, where $V \in [0, V_{\max}]$.

Proof: The above results can be proved by expanding the proof of Theorem 1 using a multi-slot drift technique [16]. We omit the proof here for brevity. ■

V. SIMULATION RESULTS

In this section, we simulate an aggregator-EVs system with parameters drawn from practical scenarios, and compare WMRA with a suboptimal greedy algorithm.

Suppose that the aggregator is connected with $N = 100$ EVs, evenly split into Type I (Ford Focus Electric) and Type II (Tesla Model S). The parameters of Type I and Type II EVs are summarized in Table I [20], [22], where $x_{i,\max}$ is derived by assuming the regulation interval $\Delta t = 5$ minutes. For example, in New England, New York, and Ontario, the common Δt is 5 minutes. Consider that each EV has random dynamics, and the system state $A_t = (G_t, e_{s,t}, e_{d,t}, \mathbf{1}_t)$ follows a finite state irreducible and aperiodic Markov chain. Specifically, at each time slot, the regulation energy amount G_t is drawn uniformly from a discrete set $\{-69.2, -69.2 + \Delta_1, -69.2 + 2\Delta_1, \dots, 69.2\}$ (kWh) with cardinality 200, where 69.2 kWh is the maximum allowed energy amount at each time slot if all N EVs are in the system. The unit costs of external sources, i.e., $e_{s,t}$ and $e_{d,t}$, are drawn uniformly from a discrete set $\{0.1, 0.1 + \Delta_2, 0.1 + 2\Delta_2, \dots, 0.12\}$ (dollars/kWh) with cardinality 200. The indicator random variable $\mathbf{1}_{i,t}$, i.e., whether the i -th EV is in the system, follows a 2-state Markov chain as

TABLE I
PARAMETERS FOR TYPE I AND TYPE II EVs

	Type I EV	Type II EV
$s_{i,\text{cap}}$ (kWh)	23	40
$x_{i,\max}$ (kWh)	0.55	0.83

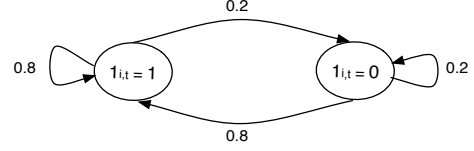


Fig. 1. Transition probabilities of $\mathbf{1}_{i,t}$, $\forall i$.

shown in Fig. 1. The returning energy state $s_{i,t_{i,r,k+1}}$ of each EV is drawn uniformly from $[s_{i,t_{i,l,k}} - \Delta_3, s_{i,t_{i,l,k}} + \Delta_3]$ with $\Delta_3 = 5\%s_{i,\text{cap}}$, and is guaranteed to be within $[s_{i,\min}, s_{i,\max}]$. We set $s_{i,\min} = 0.1s_{i,\text{cap}}$ and $s_{i,\max} = 0.9s_{i,\text{cap}}$ except otherwise mentioned. In the objective function of **P1**, we set $U(x) = \log(1 + x)$ and $\omega_i = 1, \forall i$. The battery degradation cost function of each EV is $C_i(x) = x^2$, and the upper bound $c_{i,\text{up}}$ is set to be $x_{i,\max}^2/4$.

To allocate the requested regulation amount, we apply WMRA in Algorithm 1 at each time slot. The simulation is performed over 1000 time slots. The social welfare at time slot τ is considered as the objective function of **P1** with $T = \tau$. For comparison, we consider a greedy algorithm which only optimizes the system performance at the current time slot. Thus, its regulation allocation at each time slot is derived from the following optimization problem.

$$\begin{aligned}
 & \max_{\mathbf{x}_{d,t}, \mathbf{x}_{u,t}} \left(\sum_{i=1}^N \omega_i U(x_{i,t}) \right) - e_t \\
 & \text{s.t.} \quad (3), (4), (5), (6), \text{ and} \\
 & \quad \mathbf{1}_{d,t} C_i(x_{id,t}) + \mathbf{1}_{u,t} C_i(x_{iu,t}) \leq c_{i,\text{up}}, \forall i.
 \end{aligned}$$

The above problem is a convex optimization problem, and we use the standard solver in MATLAB to obtain its solution.

In Figs. 2 and 3, we compare the performance of WMRA with $V = V_{\max}$ and the performance of the greedy algorithm. From Fig. 2, with $s_{i,\max} = 0.9s_{i,\text{cap}}$, WMRA is uniformly superior to the greedy algorithm for all times, and the advantage is about 28%. In Fig. 3, we vary $s_{i,\max}$ from $0.3s_{i,\text{cap}}$ to $0.9s_{i,\text{cap}}$. The observations are as follows. First, WMRA uniformly outperforms the greedy algorithm over different values of $s_{i,\max}$. Second, as $s_{i,\max}$ increases, the social welfare under WMRA keeps on increasing. This is because increasing $s_{i,\max}$ effectively increases V_{\max} , which improves the performance of WMRA. This observation is also consistent with the remarks after Theorem 1. In contrast, the social welfare under the greedy algorithm reaches saturation when $s_{i,\max} \geq 0.7s_{i,\text{cap}}$.

In Fig. 4, we show the performance of WMRA with different values of V as $[0.2, 0.4, 0.6, 0.8, 1, 1.5, 2]V_{\max}$. As expected, the social welfare grows as the value of V grows. Particularly, WMRA outperforms the greedy algorithm even with $V = 0.2V_{\max}$. From Lemma 6, we know that the energy

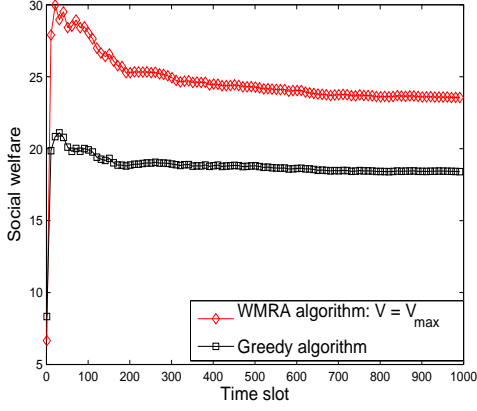


Fig. 2. Time-averaged social welfare with $V = V_{\max}$.

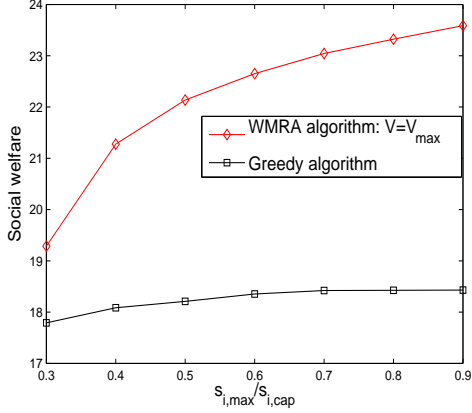


Fig. 3. Time-averaged social welfare with various $s_{i,\max}$ and $V = V_{\max}$.

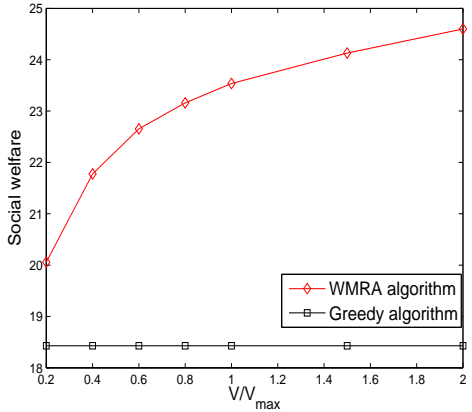


Fig. 4. Time-averaged social welfare with various values of V .

state of each EV is guaranteed to be within $[s_{i,\min}, s_{i,\max}]$ when $V \in [0, V_{\max}]$. In Fig. 5, for V being V_{\max} , $1.5V_{\max}$, and $2V_{\max}$, we show the evolutionary behaviours of a Type I EV's energy state under WMRA. We see that, when $V = V_{\max}$, the energy state is always within the preferred range; in contrast, when $V = 1.5V_{\max}$ or $2V_{\max}$, the associated energy state can exceed the preferred range from time to time. Furthermore,

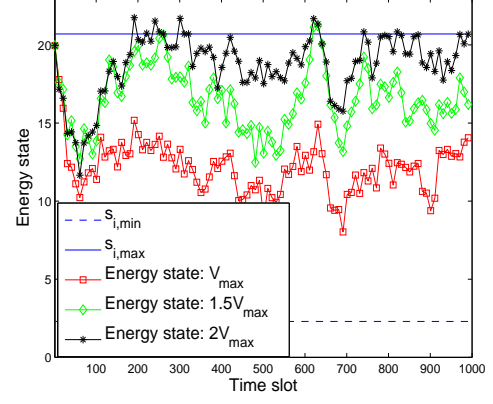


Fig. 5. Sample path of a Type I EV's energy state with $V = [1, 1.5, 2]V_{\max}$.

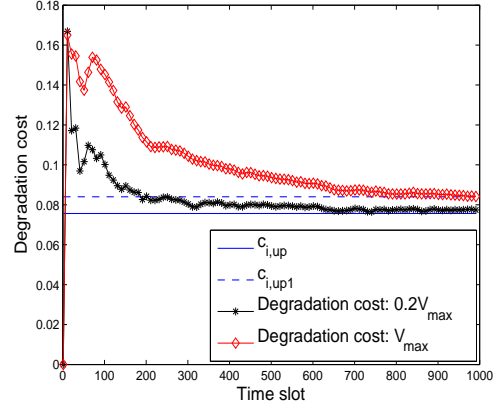


Fig. 6. Sample path of a Type I EV's time-averaged degradation cost with $V = 0.2V_{\max}$ and V_{\max} .

the larger V the more frequently such violation happens. Therefore, the observations in Figs. 4 and 5 demonstrate the significance of V_{\max} in achieving the maximum social welfare under WMRA considering the restriction of EV's battery capacity.

In Fig. 6, we display the time-averaged degradation cost of a Type I EV under WMRA with $V = 0.2V_{\max}$ and V_{\max} , respectively. We see that, first, for both values of V , the average degradation cost approaches the upper bound $c_{i,\text{up}}$ gradually, which conforms to the feasibility conclusion in Theorem 2 1) that, for each EV, the long-term constraint of the battery degradation cost (7) holds; second, the average degradation cost with smaller V arrives at the upper bound sooner. This second observation together with the observation in Fig. 4 demonstrate the role of V in the trade-off between the objective and the constraints of **P3**. Hence, in practice, if the aggregator needs to satisfy the constraint of the degradation cost within a finite operational time, it can use a smaller V , sacrificing some social welfare. Alternatively, it can also employ an operational upper bound, which is smaller than the actual upper bound, to ensure that the degradation cost constraint is met within finite time. For example, suppose that in WMRA we set $c_{i,\text{up}}$ to be only 90% of the actual upper

bound $c_{i,\text{up1}}$, as indicated in Fig. 6. Then, the actual upper bound $c_{i,\text{up1}}$ can be achieved under WMRA with $V = V_{\text{max}}$ when $T = 1000$.

VI. CONCLUSION

We studied a practical model of a dynamic aggregator-EVs system providing regulation service to a power grid. We formulated the regulation allocation optimization as a long-term time-averaged social welfare maximization problem. Our formulation accounts for random system dynamics, battery constraints, the costs of battery degradation and external energy sources, and especially, the dynamics of EVs. Adopting a general Lyapunov optimization framework, we developed a real-time WMRA algorithm for the aggregator to fairly allocate the regulation amount among EVs. The algorithm does not require any knowledge of the statistics of the system state. We were able to bound the performance of WMRA to that under the optimal solution, and showed that the performance of WMRA is asymptotically optimal as EVs' battery capacities go to infinity. Simulation demonstrated that WMRA offers substantial performance gains over a greedy algorithm that maximizes per-slot social welfare objective.

APPENDIX A PROOF OF LEMMA 1

It is easy to see that $(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*)$ is feasible for **P1**. To show that $(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}, \bar{\mathbf{z}}_t^{\text{opt}})$ is feasible for **P2**, it suffices to show that $\bar{\mathbf{z}}_t^{\text{opt}}$ satisfies (9) and (10). Using the definition of $\bar{\mathbf{z}}_t^{\text{opt}}$, (10) naturally holds. Also, since $x_{i,t}^{\text{opt}}$ lies in $[0, x_{i,\text{max}}]$, which is a closed interval, (9) holds.

We claim that

$$\begin{aligned} f_1(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}) &= f_2(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}, \bar{\mathbf{z}}_t^{\text{opt}}) \\ &\leq f_2(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*, \mathbf{z}_t^*) \\ &\leq f_1(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*) \\ &\leq f_1(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}). \end{aligned} \quad (23)$$

Using the definition of $\bar{\mathbf{z}}_t^{\text{opt}}$ in $f_2(\cdot)$, the first equality holds. The first and the third inequalities hold since $(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*, \mathbf{z}_t^*)$ and $(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}})$ are optimal for $f_2(\cdot)$ and $f_1(\cdot)$, respectively. The second inequality is derived using Jensen's inequality for concave functions. Since (23) is satisfied with equality, all inequalities in (23) turn into equalities, which indicates the equivalence of **P1** and **P2**.

APPENDIX B PROOF OF LEMMA 2

Let T be large enough. For the i -th EV, decompose the total regulation amount within $T - 1$ time slots as

$$\sum_{t=0}^{T-1} b_{i,t} = \sum_{t=0}^{t_{il,k^*}-1} b_{i,t} + \sum_{t=t_{il,k^*}}^{T-1} b_{i,t}, \quad (24)$$

where $k^* \triangleq n_i(T - 1)$. On the right hand side of (24), the first term corresponds to the total regulation amount before the last leaving time until time $T - 1$, and the second term corresponds to the rest total regulation amount.

To show (13), it suffices to show that $\lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}[\sum_{t=0}^{t_{il,k^*}-1} b_{i,t}]$ and $\lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}[\sum_{t=t_{il,k^*}}^{T-1} b_{i,t}]$ both equal zero. Note that $b_{i,t} = 0$ when $\mathbf{1}_{i,t} = 0$. Together with the boundedness of $s_{i,t}$, it is not difficult to see that the latter one equals zero. We now show that the former one also equals zero. Based on (2), the first term in (24) can be expressed as

$$\begin{aligned} \sum_{t=0}^{t_{il,k^*}-1} b_{i,t} &= \sum_{k=1}^{k^*} s_{i,t_{il,k}} - \sum_{k=1}^{k^*} s_{i,t_{ir,k}} \\ &= \sum_{k=1}^{k^*-1} \Delta_{i,k} + s_{i,t_{il,k^*}} - s_{i,t_{ir,1}}. \end{aligned} \quad (25)$$

Consider the first term on the right hand side of (25), and we have

$$\begin{aligned} 0 &\leq \left| \frac{1}{T} \mathbb{E} \left[\sum_{k=1}^{k^*-1} \Delta_{i,k} \right] \right| \\ &\leq \mathbb{E} \left[\left| \frac{1}{T} \sum_{k=1}^{k^*-1} \Delta_{i,k} \right| \right] \\ &\leq \mathbb{E} \left[\left| \frac{1}{n_i(T-1)-1} \sum_{k=1}^{n_i(T-1)-1} \Delta_{i,k} \right| \right], \end{aligned} \quad (26)$$

where in (26) we have replaced k^* with its definition and the inequality holds since $n_i(T - 1) < T - 1$. Using the assumptions that $\lim_{K \rightarrow \infty} \mathbb{E} \left[\left| \frac{1}{K} \sum_{k=1}^K \Delta_{i,k} \right| \right] = 0$ and $\lim_{t \rightarrow \infty} n_i(t) = \infty$, from (26), we have $\lim_{T \rightarrow \infty} \mathbb{E} \left[\left| \frac{1}{n_i(T-1)-1} \sum_{k=1}^{n_i(T-1)-1} \Delta_{i,k} \right| \right] = 0$, and therefore,

$$\lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{k=1}^{k^*-1} \Delta_{i,k} \right] = 0. \quad (27)$$

Taking expectations of both sides of (25), dividing them by T , then taking limits gives

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{t_{il,k^*}-1} b_{i,t} \right] &= \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{k=1}^{k^*-1} \Delta_{i,k} \right] \\ &+ \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[s_{i,t_{il,k^*}} - s_{i,t_{ir,1}} \right] = 0, \end{aligned}$$

where the last equality is derived by using (27) and the boundedness of $s_{i,t}$. This completes the proof.

APPENDIX C PROOF OF PROPOSITION 1

Based on the definition of $L(\Theta_t)$,

$$\begin{aligned} L(\Theta_{t+1}) - L(\Theta_t) &= \frac{1}{2} \sum_{i=1}^N H_{i,t+1}^2 + J_{i,t+1}^2 + K_{i,t+1}^2 - H_{i,t}^2 - J_{i,t}^2 - K_{i,t}^2. \end{aligned} \quad (28)$$

In (28), $H_{i,t+1}^2 - H_{i,t}^2$ and $J_{i,t+1}^2 - J_{i,t}^2$ can be upper bounded as follows.

$$\begin{aligned} & H_{i,t+1}^2 - H_{i,t}^2 \\ & \leq 2H_{i,t}(z_{i,t} - x_{i,t}) + x_{i,\max}^2 \end{aligned} \quad (29)$$

$$\begin{aligned} & J_{i,t+1}^2 - J_{i,t}^2 \\ & \leq 2J_{i,t}[\mathbf{1}_{d,t}C_i(x_{id,t}) + \mathbf{1}_{u,t}C_i(x_{iu,t}) - c_{i,\text{up}}] \\ & \quad + [c_{i,\text{up}}^2, (c_{i,\max} - c_{i,\text{up}})^2]^+. \end{aligned} \quad (30)$$

Now consider $K_{i,t+1}^2 - K_{i,t}^2$. When $\mathbf{1}_{i,t} = 1$, since $K_{i,t+1} = K_{i,t} + b_{i,t}$,

$$K_{i,t+1}^2 - K_{i,t}^2 \leq 2K_{i,t}b_{i,t} + x_{i,\max}^2. \quad (31)$$

When $\mathbf{1}_{i,t} = 0$, we have $b_{i,t} = 0$. Particularly, for $t \in \{t_{il,k}, t_{il,k} + 1, \dots, t_{ir,k+1} - 2\}$, $\forall k \in \{1, 2, \dots\}$, and $t \leq t_{ir,1} - 2$ (if such t is feasible), there is $K_{i,t+1} = K_{i,t}$, so, we can express

$$K_{i,t+1}^2 - K_{i,t}^2 = 2K_{i,t}b_{i,t}. \quad (32)$$

For $t = t_{ir,k} - 1, \forall k = \{1, 2, \dots\}$, there is $K_{i,t+1}^2 - K_{i,t}^2 = (s_{i,t_{ir,k}} - c_i)^2 - K_{i,t}^2 \leq c_i^2$, so,

$$K_{i,t+1}^2 - K_{i,t}^2 \leq 2K_{i,t}b_{i,t} + c_i^2. \quad (33)$$

Combining (31), (32), and (33),

$$K_{i,t+1}^2 - K_{i,t}^2 \leq 2K_{i,t}b_{i,t} + [x_{i,\max}^2, c_i^2]^+. \quad (34)$$

Imposing the upper bounds (29), (30), and (34) on the right hand side of (28), taking the conditional expectation of both sides, then subtracting the term $V\mathbb{E}\left[\sum_{i=1}^N \omega_i U(z_{i,t}) - e_t | \Theta_t\right]$ gives the upper bound in Proposition 1.

APPENDIX D PROOF OF LEMMA 4

We need the following lemma.

Lemma 7: Under the WMRA algorithm, queue backlog $H_{i,t}$ associated with the i -th EV is upper bounded as follows:

$$H_{i,t} \leq V\omega_i\mu + x_{i,\max}.$$

Proof: This can be shown using a similar method as in [16], and the technical condition (8) is needed. ■

1) Consider $G_t > 0$. Suppose that when $K_{i,t} > x_{i,\max} + V(\omega_i\mu + e_{\max})$, one optimal solution under WMRA is $\tilde{\mathbf{x}}_{d,t}$ with $\tilde{x}_{id,t} > 0$. Then we show that we can find another solution with $\tilde{x}_{jd,t}, \forall j \neq i$ and $\tilde{x}_{id,t} = 0$ resulting in a strictly smaller objective value, which is a contradiction.

Using the objective function of **b1**, this is equivalent to show

$$\begin{aligned} & V e_{s,t} \left[G_t - \sum_{i=1}^N \tilde{x}_{id,t} \right] - \sum_{i=1}^N H_{i,t} \tilde{x}_{id,t} \\ & + \sum_{i=1}^N J_{i,t} C_i(\tilde{x}_{id,t}) + \sum_{i=1}^N K_{i,t} \tilde{x}_{id,t} \\ & > V e_{s,t} \left[G_t - \sum_{i=1}^N \tilde{x}_{id,t} + \tilde{x}_{id,t} \right] - \sum_{j \neq i} H_{j,t} \tilde{x}_{jd,t} \\ & + \sum_{j \neq i} J_{j,t} C_j(\tilde{x}_{jd,t}) + \sum_{j \neq i} K_{j,t} \tilde{x}_{jd,t}, \end{aligned}$$

which is equivalent to

$$-H_{i,t} \tilde{x}_{id,t} + J_i C_i(\tilde{x}_{id,t}) + K_{i,t} \tilde{x}_{id,t} > V e_{s,t} \tilde{x}_{id,t}. \quad (35)$$

Since $J_i C_i(\tilde{x}_{id,t}) \geq 0$, from (35), it suffices to show that

$$(K_{i,t} - H_{i,t} - V e_{s,t}) \tilde{x}_{id,t} > 0. \quad (36)$$

Since $\tilde{x}_{id,t} > 0$, (36) is true by using the assumption that $K_{i,t} > x_{i,\max} + V(\omega_i\mu + e_{\max})$ and Lemma 7 in which $H_{i,t}$ is upper bounded.

2) Consider $G_t < 0$. Suppose that when $K_{i,t} < -x_{i,\max} - V(\omega_i\mu + e_{\max})$, one optimal solution under WMRA is $\tilde{\mathbf{x}}_{u,t}$ with $\tilde{x}_{iu,t} > 0$. Then there is a contradiction since we can construct another solution with $\tilde{x}_{ju,t}, \forall j \neq i$ and $\tilde{x}_{iu,t} = 0$ which results in a strictly smaller objective value. The proof is similar as that in 1) and is omitted here.

APPENDIX E PROOF OF THEOREM 1

We first give the following fact, which is a direct consequence of the results in [16].

Lemma 8: There exists a stationary randomized regulation allocation solution $(\mathbf{x}_{d,t}^s, \mathbf{x}_{u,t}^s)$ that only depends on system state A_t , and there are

$$\mathbb{E}[x_{i,t}^s] = z_i^s, \forall i, \text{ for some } z_i^s \in [0, x_{i,\max}], \quad (37)$$

$$\mathbb{E}[e_t^s] - \sum_{i=1}^N \omega_i U(z_i^s) \leq -f_2(\hat{\mathbf{x}}_{d,t}, \hat{\mathbf{x}}_{u,t}, \hat{\mathbf{z}}_t), \quad (38)$$

$$\mathbb{E}[\mathbf{1}_{d,t} C_i(x_{id,t}^s) + \mathbf{1}_{u,t} C_i(x_{iu,t}^s)] \leq c_{i,\text{up}}, \forall i, \text{ and } \quad (39)$$

$$\mathbb{E}[b_{i,t}^s] = 0, \forall i, \quad (40)$$

where the expectations are taken over the randomness of the system and the randomness of $(\mathbf{x}_{d,t}^s, \mathbf{x}_{u,t}^s)$, and $(\hat{\mathbf{x}}_{d,t}, \hat{\mathbf{x}}_{u,t}, \hat{\mathbf{z}}_t)$ is an optimal solution for **P3**.

1) For brevity, define $W_t \triangleq \left(\sum_{i=1}^N \omega_i U(z_{i,t})\right) - e_t$. Since WMRA minimizes the upper bound in (22), plug $(\mathbf{x}_{d,t}^s, \mathbf{x}_{u,t}^s)$ on the right hand side of (22) together with $z_{i,t} = z_i^s, \forall t$, we have

$$\Delta(\Theta_t) - V\mathbb{E}[\tilde{W}_t | \Theta_t] \leq B - V f_2(\hat{\mathbf{x}}_{d,t}, \hat{\mathbf{x}}_{u,t}, \hat{\mathbf{z}}_t), \quad (41)$$

where (37), (38), (39), and (40) are used. Since $\tilde{W}_t \leq \sum_{i=1}^N \omega_i U(x_{i,\max})$, from (41),

$$\Delta(\Theta_t) \leq D \triangleq B + V \left(\sum_{i=1}^N \omega_i U(x_{i,\max}) - f_2(\hat{\mathbf{x}}_{d,t}, \hat{\mathbf{x}}_{u,t}, \hat{\mathbf{z}}_t) \right).$$

Using Theorem 4.1 in [16], $\mathbb{E}[H_{i,t}]$ and $\mathbb{E}[J_{i,t}]$ are upper bounded by $\sqrt{2tD + 2L(\Theta_0)}, \forall t$. Hence, the virtual queues $H_{i,t}$ and $J_{i,t}$ are mean rate stable and the following limit constraints hold.

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{z}_{i,t}] = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{x}_{i,t}], \forall i, \quad (42)$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\mathbf{1}_{d,t} C_i(\tilde{x}_{id,t}) + \mathbf{1}_{u,t} C_i(\tilde{x}_{iu,t})] \leq c_{i,\text{up}}, \forall i.$$

Since $s_{i,t}$ is bounded under WMRA by Lemma 6, using Lemma 2, we have $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{b}_{i,t}] = 0, \forall i$. In

addition, note that $(\tilde{\mathbf{x}}_{d,t}, \tilde{\mathbf{x}}_{u,t})$ is derived under the constraints of the optimization problems (a), (b1), and (b2). Therefore, we have that $(\tilde{\mathbf{x}}_{d,t}, \tilde{\mathbf{x}}_{u,t})$ is feasible for **P3**, **P2**, and **P1**.

2) Taking expectations of both sides of (41) and summing over $t \in \{0, 1, \dots, T-1\}$ for some $T > 1$, we have

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{W}_t] \geq \frac{\mathbb{E}[L(\Theta_t) - L(\Theta_0)]}{VT} + f_2(\hat{\mathbf{x}}_{d,t}, \hat{\mathbf{x}}_{u,t}, \hat{\mathbf{z}}_t) - B/V$$

$$\geq f_2(\hat{\mathbf{x}}_{d,t}, \hat{\mathbf{x}}_{u,t}, \hat{\mathbf{z}}_t) - B/V - \mathbb{E}[L(\Theta_0)]/VT, \quad (43)$$

where (43) holds since $L(\Theta_t)$ is non-negative. Also,

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{W}_t] = \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \left[\left(\sum_{i=1}^N \omega_i U(\tilde{z}_{i,t}) \right) - \tilde{c}_t \right]$$

$$\leq \sum_{i=1}^N \omega_i U \left(\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{z}_{i,t}] \right) - \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{c}_t], \quad (44)$$

where the inequality in (44) is derived using Jensen's inequality for concave functions. Combining (43) and (44) and taking limits on both sides, there is

$$\sum_{i=1}^N \omega_i U \left(\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{z}_{i,t}] \right) - \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{c}_t]$$

$$\geq f_2(\hat{\mathbf{x}}_{d,t}, \hat{\mathbf{x}}_{u,t}, \hat{\mathbf{z}}_t) - B/V \quad (45)$$

$$\geq f_2(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*, \mathbf{z}_t^*) - B/V \quad (46)$$

$$= f_1(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}) - B/V, \quad (47)$$

where $(\mathbf{x}_{d,t}^*, \mathbf{x}_{u,t}^*, \mathbf{z}_t^*)$ and $(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}})$ are defined in Section III-A, (45) holds since $\mathbb{E}[L(\Theta_0)]$ is bounded, (46) holds since the feasible set of the optimization variables is enlarged from **P2** to **P3**, and (47) is true due to Lemma 1.

Rewrite the objective function of **P1** under WMRA, i.e., $f_1(\tilde{\mathbf{x}}_{d,t}, \tilde{\mathbf{x}}_{u,t})$, as

$$\sum_{i=1}^N \omega_i U \left(\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{z}_{i,t}] \right) - \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{c}_t]$$

$$+ \sum_{i=1}^N \omega_i U \left(\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{x}_{i,t}] \right)$$

$$- \sum_{i=1}^N \omega_i U \left(\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\tilde{z}_{i,t}] \right).$$

Due to (42), the last two terms cancel each other. Hence, by (47), we have $f_1(\tilde{\mathbf{x}}_{d,t}, \tilde{\mathbf{x}}_{u,t}) \geq f_1(\mathbf{x}_{d,t}^{\text{opt}}, \mathbf{x}_{u,t}^{\text{opt}}) - B/V$, which completes the proof.

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